

# Programming Chemical Systems

The background of the slide is a complex, abstract pattern of overlapping, semi-transparent shapes. These shapes are primarily teardrop or oval-like in form, arranged in a dense, repeating pattern that creates a sense of depth and movement. The color palette is diverse, including shades of purple, blue, green, yellow, orange, red, and brown. The overall effect is a vibrant, multi-layered visual texture that frames the text.

Luca Cardelli, University of Oxford  
Berkeley Programming Systems Seminar, 2022-09-26

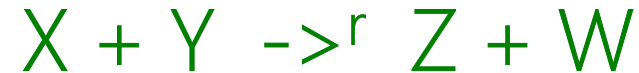
# Outline

1. From "any" Digital or Analog system to a Chemical Reaction Network
2. From (made-up) Chemical Reaction Networks to (real) Molecules that implement them (skipped)
3. Languages for Chemical Reaction Networks

# Part 1

From (almost) any algorithm  
and (almost) any dynamical system  
to a Chemical Reaction Network

# Chemical Reaction Networks (CRN)



- A *phenomenological model* of kinetics in the natural sciences  
By (only) observing naturally occurring reactions
- A *programming language*, *finitely* encoded in the genome  
By which living things manage the *unbounded* processing of matter and information
- A *mathematical structure*, rediscovered in many forms  
Vector Addition Systems, Petri Nets, Bounded Context-Free Languages, Population Protocols, ...
- A description of *mechanism* (“instructions” / “interactions”) rather than *behavior* (“equations” / “approximations”)  
Although the two are related in precise ways  
Enabling, e.g., the study of the evolution of *mechanism* through unchanging *behavior*

# Part 1a

“Digital” computation = algorithms

# Programming Examples

*spec*

$$Y := 2X$$

$$Y := \lfloor X/2 \rfloor$$

$$Y := X1 + X2$$

$$Y := \min(X1, X2)$$

*program*

$$X \rightarrow Y + Y$$

$$X + X \rightarrow Y$$

$$X1 \rightarrow Y$$

$$X2 \rightarrow Y$$

$$X1 + X2 \rightarrow Y$$

# Advanced Programming Examples

*spec*

$Y := \max(X1, X2)$

*program*

$X1 \rightarrow L1 + Y$

$X2 \rightarrow L2 + Y$

$L1 + L2 \rightarrow K$

$Y + K \rightarrow 0$

$\max(X1, X2) =$   
 $(X1 + X2) - \min(X1, X2)$

(but is not computed  
"sequentially")

Approximate Majority

$(X, Y) :=$

if  $X \geq Y$  then  $(X + Y, 0)$

if  $Y \geq X$  then  $(0, X + Y)$

$X + Y \rightarrow Y + B$

$Y + X \rightarrow X + B$

$B + X \rightarrow X + X$

$B + Y \rightarrow Y + Y$

# CRN Semantics (discrete state space)\*

- No-time (concurrent) semantics
  - Ignore rates. The multisets of molecules are rewritten according to the reactions, which may fire concurrently when not in resource conflict. This results in a **Petri Net**.
- Discrete time semantics
  - Reaction rates determine the *probability* with which reactions fire at discrete time intervals, then they behave as multiset rewrites at each discrete time interval. This results in a **Discrete Time Markov Chain (DTMC)**.
- Continuous time semantics of CRNs
  - Reaction rates determine the *propensity* with which reactions fire (both the probability of firing and the inter-firing intervals), then they behave as multiset rewrites. This results in a **Continuous Time Markov Chain (CTMC)**.
  - These CRNs are called **FSCRN (finite stochastic CRN)**.

\*Discrete state space means that each chemical species has a number of molecules (a nonnegative integer); then time can be modeled as one of the above.



# Programming <sup>"approximately"</sup> *any* algorithm as a FSCRN

A FSCRN is a *finite* set of reactions over a *finite* set of species

FSCRNs are not Turing complete

Like Petri nets: reachability is decidable

But unlike Petri nets, FSCRNs are *approximately* Turing complete

Because reactions have also *rates*

This make it possible to approximate Turing completeness by approximating test-for-zero in a register machine.

The probability of error (in test-for-zero) can be made arbitrarily small over the entire (undecidably long) computation.

Computation with Finite Stochastic Chemical Reaction Networks

David Soloveichik\* Matthew Cook<sup>†</sup> Erik Winfree<sup>‡</sup> Jehoshua Bruck<sup>§</sup>

Adding polymerization to the model makes it fully Turing complete  
but the syntax becomes considerably more complex

Formal Molecular Biology

Vincent Danos\* Cosimo Laneve<sup>†</sup>

# Register Machines (almost...)

i: INC  $R_1$ ; JMP j

$$PC_i \rightarrow R_1 + PC_j$$

i: DEC  $R_1$ ; JMP j

$$PC_i + R_1 \rightarrow PC_j$$

i: IF  $R_2 > 0$  {INC  $R_1$ ; JMP j}

$$PC_i + R_2 \rightarrow R_2 + R_1 + PC_j$$

i: IF  $R_2 = 0$  ...

??? Whatever trick we use will have some error

- Turing-complete up to an arbitrarily small error
  - The error bound is set in advance uniformly for any computation of arbitrary length (because we cannot know how long the computation will last), and the machine will progressively “slow down” to always stay below that bound.

■ David Soloveichik, Matt Cook, Erik Winfree, Shuki Bruck, "Computation with Finite Stochastic Chemical Reaction Networks".  
[ [Natural Computing, \(online Feb 2008\)](#), or [Technical Report: CaltechPARADISE:2007.ETR085: .pdf](#) ]

# CRN Semantics (continuous state space/deterministic)\*

- ODE semantics of CRNs

- The chemical *Law of Mass Action* says that the *flux* of a reaction is determined by the product of the concentrations of the reagents, times the reaction rate.

**Definition** (CRN Flux) Let  $(\mathcal{A}, \mathcal{R})$  be a CRN. Let  $F(V, T) \in \mathbb{R}_{\geq 0}^{|\mathcal{A}|} \rightarrow \mathbb{R}^{|\mathcal{A}|}$  be the flux of the CRN at volume  $V \in \mathbb{R}_{> 0}$  and temperature  $T \in \mathbb{R}_{\geq 0}$ . For a concentration vector  $\mu \in \mathbb{R}_{\geq 0}^{|\mathcal{A}|}$  we assume  $F(V, T)(\mu) = \sum_{\tau \in \mathcal{R}} v_{\tau} \alpha_{\tau}(V, T, \mu)$  with stoichiometric vector  $v_{\tau}$  and rate function  $\alpha_{\tau}$ .

**Law of Mass Action**  $F(V, T)$  makes up the r.h.s. of an ODE system  $\partial \mathcal{A} = F(V, T)$

**State produced by a CRN**  $\mathcal{C} = (\mathcal{A}, \mathcal{R})$  (species  $\mathcal{A}$ , reactions  $\mathcal{R}$ )

with flux  $F$  (r.h.s. of its mass action ODEs) at time  $t$ ,

from initial state  $(x_0, V, T)$  (initial concentrations  $x_0$ , volume  $V$ , temperature  $T$ ):

$$\llbracket ((\mathcal{A}, \mathcal{R}, x_0), V, T) \rrbracket (H)(t) = (G(t), V, T)$$

$$\text{let } G : [0 \dots H) \rightarrow \mathbb{R}^{|\mathcal{A}|} \text{ be the solution of } G(t') = x_0 + \int_0^{t'} F(V, T)(G(s)) ds$$

\*Continuous state space means each chemical species has a concentration (a real number); concentrations are approximations of the number of molecules via the Avogadro constant.

# CRN Semantics (continuous state space/stochastic)\*

- CME semantics of CRNs (Chemical Master Equation)
  - Kolmogorov forward equation of the Markov Chain produced by the CRN.
  - Unfeasible to solve or even simulate (to compute the distribution of outcomes)
  - The Gillespie algorithm produces individual samples (traces) of the CME distribution
- LNA semantics of CRNs (Linear Noise Approximation)

Gaussian state (mean & variance) produced by a CRN  $\mathcal{C} = (\mathcal{A}, \mathcal{R})$  (species  $\mathcal{A}$ , reactions  $\mathcal{R}$ ) with flux  $F$  (r.h.s. of its mass action ODEs) at time  $t$ , with  $\boldsymbol{\mu}_\mu(0) = \boldsymbol{\mu}$  and  $\boldsymbol{\Sigma}_{\mu, \Sigma}(0) = \boldsymbol{\Sigma}$ .

$$\llbracket ((\mathcal{A}, \mathcal{R}, x_0), V, T) \rrbracket (H)(t) = (\boldsymbol{\mu}_\mu(t), \boldsymbol{\Sigma}_{\mu, \Sigma}(t), V, T)$$

$$\boldsymbol{\mu}_\mu(t) = \boldsymbol{\mu} + \int_0^t F(V, T)(\boldsymbol{\mu}_\mu(s)) ds$$

$F(V, T)(\boldsymbol{\mu}) = \sum_{\tau \in \mathcal{R}} v_\tau \alpha_\tau(V, T, \boldsymbol{\mu})$ , with stoichiometric vector  $v_\tau$  and rate function  $\alpha_\tau$ . We call  $J_F$  the Jacobian of  $F(V, T)$ , and  $J_F^\top$  its transpose. Further, define  $W(V, T)(\boldsymbol{\mu}) = \sum_{\tau \in \mathcal{R}} v_\tau v_\tau^\top \alpha_\tau(V, T, \boldsymbol{\mu})$

$$\boldsymbol{\Sigma}_{\mu, \Sigma}(t) = \boldsymbol{\Sigma} + \int_0^t J_F(\boldsymbol{\mu}_\mu(s)) \boldsymbol{\Sigma}_{\mu, \Sigma}(s) + \boldsymbol{\Sigma}_{\mu, \Sigma}(s) J_F^\top(\boldsymbol{\mu}_\mu(s)) + W(V, T)(\boldsymbol{\mu}_\mu(s)) ds$$

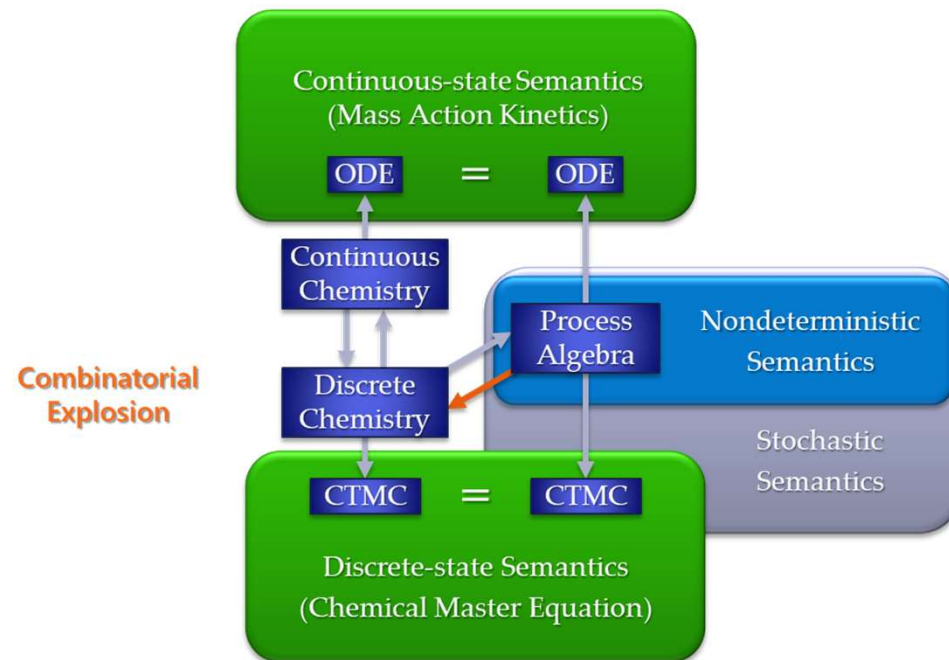
A Language for Modeling And Optimizing Experimental Biological Protocols

Luca Cardelli<sup>1</sup>, Marta Kwiatkowska<sup>1</sup> and Luca Laurenti<sup>1,2</sup>

\*Continuous state space means each species has a concentration (a real number); concentrations are an approximation of the number of molecules via the Avogadro constant.

# Chemistry as a Concurrent Language

- A connection with the theory of concurrency
  - Via Process Algebra and Petri Nets



# Finally, Some *Bad Bad* Programs



Violates thermodynamics.

(No biggie, assume there is a tiny reverse reaction.)



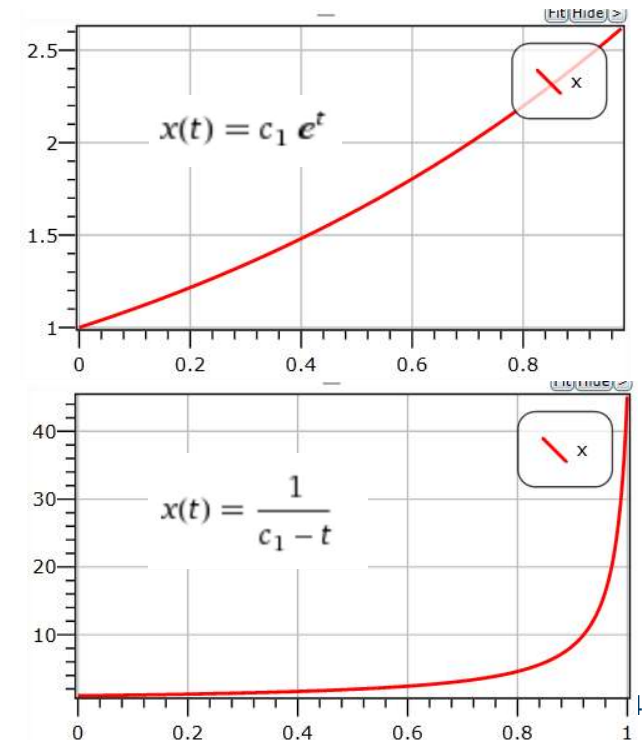
Violates conservation of mass.

(No biggie, assume there is inflow/outflow.)



Violates finite density.

(This is *really* bad.)



# Part 1b

“Analog” computation = dynamical system

# “Elementary” (NOT!) dynamical systems

A *dynamical systems* is anything characterized by a system of differential equations (ODEs).

*Elementary dynamical systems* are those that include on the r.h.s. only polynomials, trigonometry, exponentials, fractions, and their inverses.

E.g., *physics*: the equation of the simple pendulum has trigonometry on the r.h.s.:

$$\partial^2\theta = -g/l \sin(\theta)$$

E.g., *biology*: the enzyme kinetics equation has fractions on the r.h.s.:

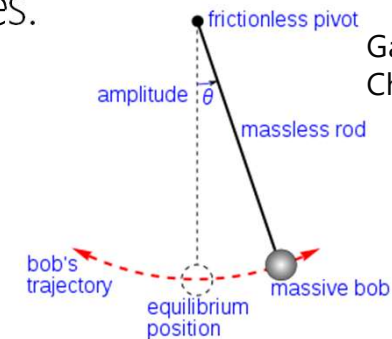
$$\partial[P] = V_{\max} [S] / (K_M + [S])$$

E.g., *metereology*: the chaotic Lorenz attractor has just 3 polynomial equations:

$$\partial x = ay - ax \quad \partial y = cx - xz - y \quad \partial z = xy - bz$$

E.g., *chemistry*: the law of mass action for CRNs implies that their ODEs are

(a restricted “Hungarian” class) of polynomials



<https://en.wikipedia.org/wiki/Pendulum>

**STEP 1, Polynomization:** All elementary ODEs can be exactly reduced to polynomial ODEs.

MATHEMATICAL THEORY OF THE DIFFERENTIAL  
ANALYZER

BY CLAUDE E. SHANNON

Abstraction of Elementary Hybrid Systems by Variable  
Transformation

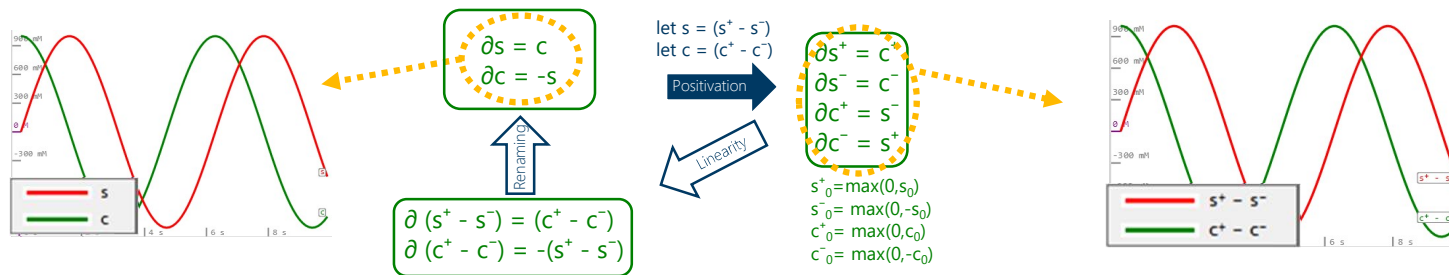
Jiang Liu<sup>1</sup>, Naijun Zhan<sup>2</sup>, Hengjun Zhao<sup>1</sup>, and Liang Zou<sup>2</sup>



"elementary"

# Programming *any* dynamical system as a CRN

Consider *the* canonical polynomial oscillator: sine/cosine



A very simple *elementary* ODE system.

But variables go negative: we can't have that in a CRN (no negative concentrations).

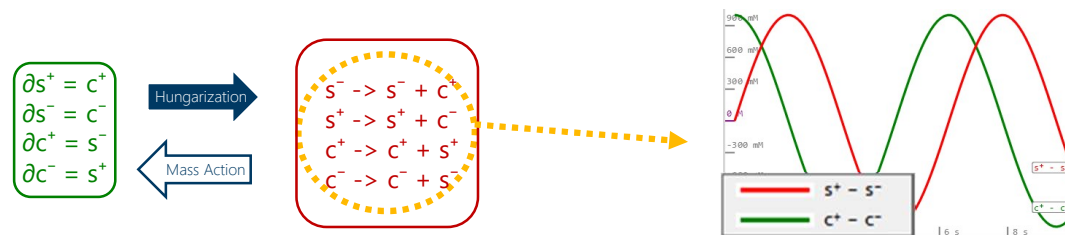
**STEP 2, Positivation:** Split potentially negative variables of polynomial ODEs into the difference of two positive variables. Obtain the same trajectories as differences.

**Biomolecular implementation of linear I/O systems**

K. Oishi E. Klavins

# Programming <sup>"elementary"</sup> *any* dynamical system as a CRN

Translate positive ODEs to chemical reactions



The Law of Mass Action tells us how to produce polynomial ODEs from CRNs. The inverse process is called Hungarization, it works for *Hungarian* ODEs (polynomial ODEs where each negative monomial has the l.h.s. differentiated variable as a factor).

**STEP 3, Hungarization:** Translate polynomial ODEs to chemical reaction networks: each monomial on the r.h.s. produces one reaction.

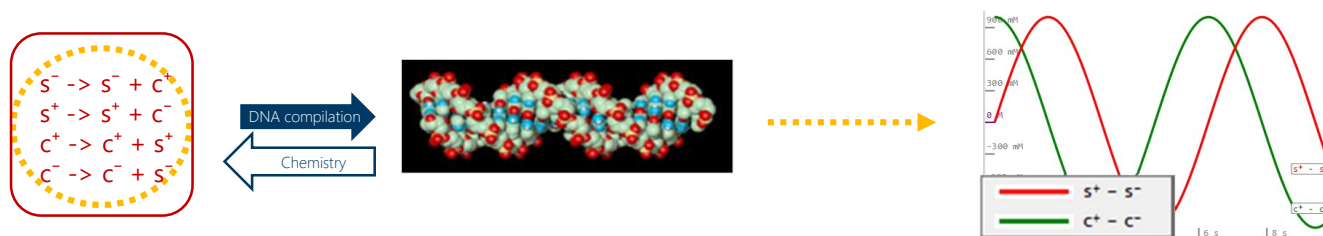
ON THE INVERSE PROBLEM OF REACTION KINETICS  
V. HÁRS - J. TÓTH

Subject to the ODEs being *Hungarian*, but that is always satisfied after positivation!

E.g. the Lorenz chaotic attractor is already polynomial but not Hungarian, it cannot be translated to mass action reactions without first doing positivation.

# Programming *any*<sup>"elementary"</sup> dynamical system as a CRN

Translate those CRNs to (real, DNA) molecules



Chemistry tells us (sometimes) what reactions molecules obey.

The inverse process is possible for DNA molecules, because we can "program" them.

**STEP 4, Molecular programming:** Translate any mass action chemical reaction network into a set of DNA molecules that obey those reactions.

Works up to an arbitrarily good approximation of Mass Action kinetics, and up to time rescaling.

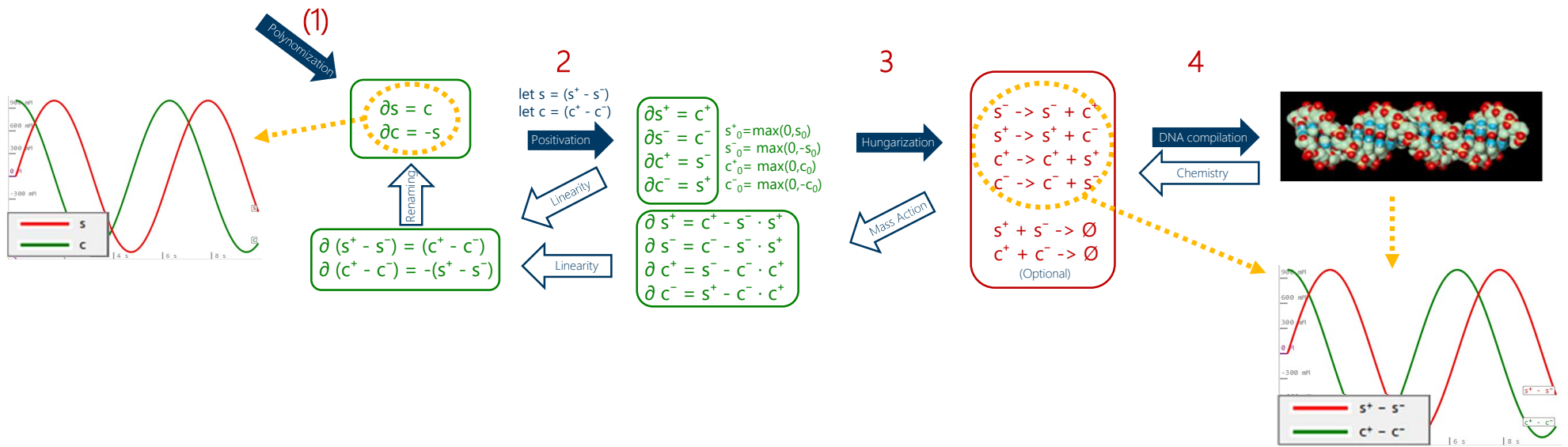
DNA as a universal substrate for chemical kinetics

David Soloveichik, Georg Seelig, and Erik Winfree  
PNAS March 23, 2010 107 (12) 5393-5398; <https://doi.org/10.1073/pnas.0909380107>

"elementary"

# Programming *any* dynamical system as a CRN

Thus, CNRs are "Shannon complete", and can be physically realized



# Summarizing

- Chemistry is (also) a formal language that we can use to implement *any* algorithm and *any* dynamical system with *real* (DNA) molecules
- Turing complete and “Shannon complete”
- ANY collection of abstract chemical reactions can be implemented with specially designed DNA molecules, with accurate kinetics (up to time scaling).
- Approaching a situation where we can “systematically compile” (synthesize) a model to DNA molecules, run an (automated) protocol, and observe (sequence) the results in a closed loop.

Part 3

Languages for CRNs

# Obviously...

Yes of course, there are CRN packages in Python, Matlab, Mathematica, etc. etc.

Yes of course, there are scripting languages, and even operating systems, for all kinds of lab equipment and for Digital Microfluidics, like PurpleDrop [Stephenson et al. 2020]

Yes of course, there are domain specific languages like CRN++ [Vasic et al. 2018]

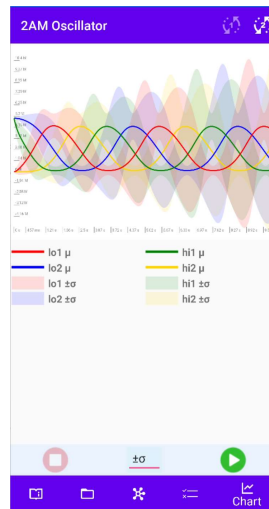
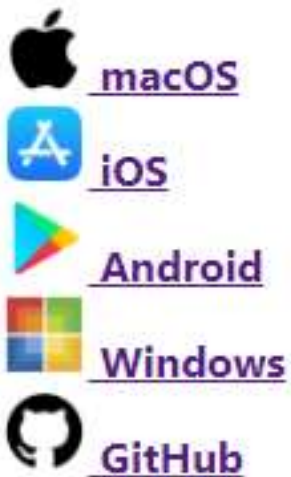
I wanted to investigate “closing the loop” between mathematical modeling and lab protocols, based on a language for CRNs.

CMSB'2020 Best Tool Paper Award

Kaemika\* app  
Integrating protocols and chemical simulation

# Kaemika

*/'kimika/*



Search "Kaemika" in the app stores  
<http://lucacardelli.name/kaemika.html>



computation



Article

## A Language for Modeling and Optimizing Experimental Biological Protocols

Luca Cardelli <sup>\*</sup>, Marta Kwiatkowska and Luca Laurenti <sup>†</sup>

An integrated language for  
chemical models & experimental protocols

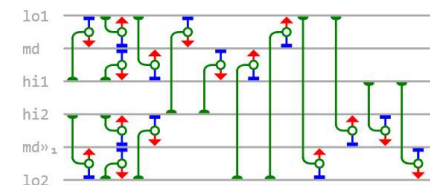
Deterministic (ODE) and  
stochastic (LNA) simulation

Chemical reaction networks (CRNs)  
and liquid-handling protocols

Reaction scores

Functional scripting

GUI





# Main features

- *Species and reactions*
  - Characterized by initial values and rates
- *"Samples" (compartments) and Protocols*
  - Isolate species and reactions in a compartment, and mix compartments
- *Kinetics (simulation)*
  - Deterministic (ODE) or stochastic (LNA) for chemical models
  - Digital microfluidics for chemical protocols
- *Programming abstractions*
  - Assemble models and protocols as compositions of modules

# Species and Reactions

```
//=====
// Lotka 1920, Volterra 1926
// (simplified with all rates = 1)
//=====

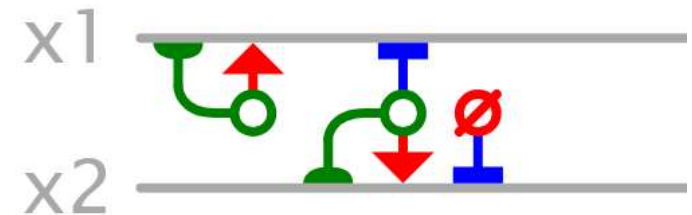
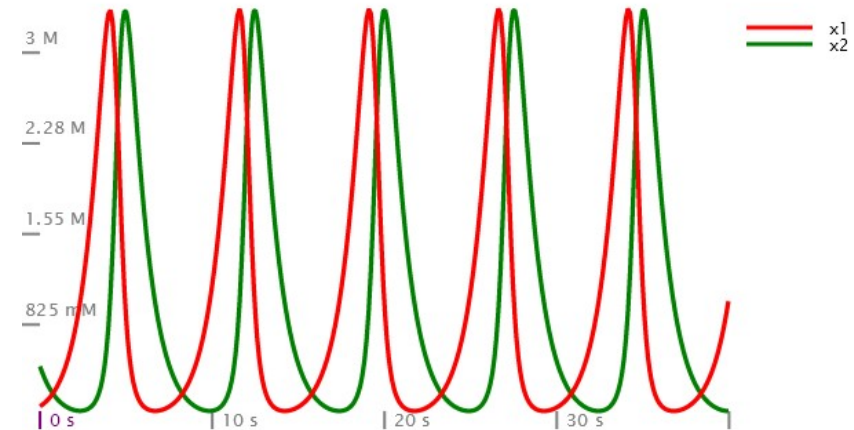
number x1_0 <- uniform(0,1) // random x1_0
number x2_0 <- uniform(0,1) // random x2_0

species x1 @ x1_0 M // prey
species x2 @ x2_0 M // predator

x1 -> x1 + x1 {1} // prey reproduces
x1 + x2 -> x2 + x2 {1} // predator eats prey
x2 -> ∅ {1} // predator dies

equilibrate for 40
```

UNDAMPED OSCILLATIONS, ETC. 1595  
UNDAMPED OSCILLATIONS DERIVED FROM THE LAW OF MASS ACTION.  
BY ALFRED J. LOTKA.  
Received June 2, 1920.



# Stochastic (LNA) simulation

- For *all* programs (any CRN, any Protocol)

## 2AM Oscillator

$$\begin{aligned} \partial lo1 &= -hi1 \cdot lo1 - 0.5 \cdot hi2 \cdot lo1 + lo1 \cdot md + 0.5 \cdot lo2 \cdot md \\ \partial hi2 &= -0.5 \cdot hi1 \cdot hi2 - hi2 \cdot lo2 + hi2 \cdot md_{\gg 1} + 0.5 \cdot lo1 \cdot md_{\gg 1} \\ \partial lo2 &= 0.5 \cdot hi1 \cdot md_{\gg 1} - hi2 \cdot lo2 - 0.5 \cdot lo1 \cdot lo2 + lo2 \cdot md_{\gg 1} \\ \partial hi1 &= -hi1 \cdot lo1 - 0.5 \cdot hi1 \cdot lo2 + hi1 \cdot md + 0.5 \cdot hi2 \cdot md \\ \partial md &= 2 \cdot hi1 \cdot lo1 + 0.5 \cdot hi1 \cdot lo2 + 0.5 \cdot hi2 \cdot lo1 - hi1 \cdot md - 0.5 \cdot hi2 \cdot md - lo1 \cdot md - 0.5 \cdot lo2 \cdot md \\ \partial md_{\gg 1} &= 0.5 \cdot hi1 \cdot hi2 - 0.5 \cdot hi1 \cdot md_{\gg 1} + 2 \cdot hi2 \cdot lo2 + 0.5 \cdot lo1 \cdot lo2 - hi2 \cdot md_{\gg 1} - 0.5 \cdot lo1 \cdot md_{\gg 1} - lo2 \cdot md_{\gg 1} \end{aligned}$$

$$\begin{aligned} \partial var(lo1) &= -cov(hi1, lo1) \cdot lo1 - 0.5 \cdot cov(hi2, lo1) \cdot lo1 - cov(lo1, hi1) \cdot lo1 - 0.5 \cdot cov(lo1, hi2) \cdot lo1 + cov(lo1, md) \cdot lo1 + hi1 \cdot lo1 + 0.5 \cdot hi2 \cdot lo1 + 0.5 \cdot cov(lo1, md) \cdot lo2 + cov(md, lo1) \cdot lo1 + 0.5 \cdot \\ &cov(md, lo1) \cdot lo2 + 0.5 \cdot cov(lo1, lo2) \cdot md + 0.5 \cdot cov(lo2, lo1) \cdot md + lo1 \cdot md + 0.5 \cdot lo2 \cdot md - 2 \cdot hi1 \cdot var(lo1) - hi2 \cdot var(lo1) + 2 \cdot md \cdot var(lo1) \end{aligned}$$

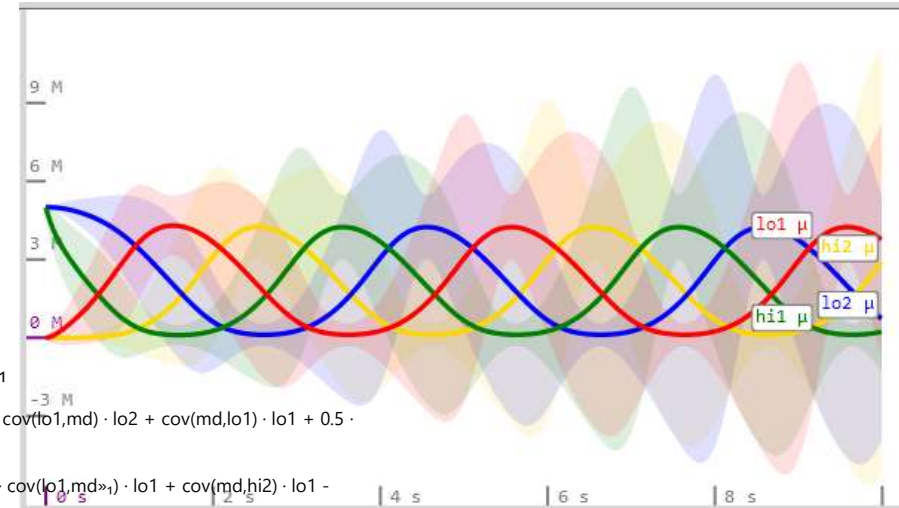
$$\begin{aligned} \partial cov(lo1, hi2) &= cov(lo1, md_{\gg 1}) \cdot hi2 - 0.5 \cdot cov(lo1, hi1) \cdot hi2 - cov(hi1, hi2) \cdot lo1 - 1.5 \cdot cov(lo1, hi2) \cdot hi1 - 0.5 \cdot cov(lo1, hi2) \cdot hi2 - cov(lo1, lo2) \cdot hi2 + 0.5 \cdot cov(lo1, md_{\gg 1}) \cdot lo1 + cov(md, hi2) \cdot lo1 - \\ &cov(lo1, hi2) \cdot lo2 + 0.5 \cdot cov(md, hi2) \cdot lo2 + cov(lo1, hi2) \cdot md + 0.5 \cdot cov(lo2, hi2) \cdot md + cov(lo1, hi2) \cdot md_{\gg 1} - 0.5 \cdot lo1 \cdot var(hi2) + 0.5 \cdot md_{\gg 1} \cdot var(lo1) \end{aligned}$$

$$\begin{aligned} \partial cov(lo1, lo2) &= 0.5 \cdot cov(lo1, md_{\gg 1}) \cdot hi1 - cov(hi1, lo2) \cdot lo1 - 0.5 \cdot cov(hi2, lo2) \cdot lo1 + cov(lo1, md_{\gg 1}) \cdot lo2 + cov(md, lo2) \cdot lo1 + 0.5 \cdot cov(md, lo2) \cdot lo2 + 0.5 \cdot cov(lo1, hi1) \cdot md_{\gg 1} - 0.5 \cdot cov(lo1, lo2) \cdot \\ &lo1 - cov(lo1, hi2) \cdot lo2 - cov(lo1, lo2) \cdot hi1 - 1.5 \cdot cov(lo1, lo2) \cdot hi2 + cov(lo1, lo2) \cdot md + cov(lo1, lo2) \cdot md_{\gg 1} - 0.5 \cdot lo2 \cdot var(lo1) + 0.5 \cdot md \cdot var(lo2) \end{aligned}$$

$$\begin{aligned} \partial cov(lo1, hi1) &= cov(lo1, md) \cdot hi1 + 0.5 \cdot cov(lo1, md) \cdot hi2 - cov(lo1, hi1) \cdot lo1 + cov(md, hi1) \cdot lo1 - 0.5 \cdot cov(lo1, hi1) \cdot lo2 - 0.5 \cdot cov(lo1, lo2) \cdot hi1 - 0.5 \cdot cov(hi2, hi1) \cdot lo1 - cov(lo1, hi1) \cdot hi1 - 0.5 \cdot \\ &cov(lo1, hi1) \cdot hi2 + 0.5 \cdot cov(md, hi1) \cdot lo2 + 2 \cdot cov(lo1, hi1) \cdot md + 0.5 \cdot cov(lo1, hi2) \cdot md + 0.5 \cdot cov(lo2, hi1) \cdot md - lo1 \cdot var(hi1) - hi1 \cdot var(lo1) \end{aligned}$$

$$\begin{aligned} \partial cov(lo1, md) &= 2 \cdot cov(lo1, hi1) \cdot lo1 - cov(hi1, md) \cdot lo1 - cov(lo1, md) \cdot lo1 - hi1 \cdot lo1 - 0.5 \cdot hi2 \cdot lo1 + 0.5 \cdot cov(lo1, hi1) \cdot lo2 - 0.5 \cdot cov(lo1, md) \cdot lo2 - cov(lo1, hi1) \cdot md + 0.5 \cdot cov(lo1, lo2) \cdot hi1 - 0.5 \cdot \\ &cov(hi2, md) \cdot lo1 + 0.5 \cdot cov(lo1, hi2) \cdot lo1 - 0.5 \cdot cov(lo1, hi2) \cdot md - 0.5 \cdot cov(lo1, lo2) \cdot md - 2 \cdot cov(lo1, md) \cdot hi1 - cov(lo1, md) \cdot hi2 + cov(lo1, md) \cdot md + 0.5 \cdot cov(lo2, md) \cdot md - lo1 \cdot md - 0.5 \cdot lo2 \cdot \\ &md + 2 \cdot hi1 \cdot var(lo1) + 0.5 \cdot hi2 \cdot var(lo1) + lo1 \cdot var(md) + 0.5 \cdot lo2 \cdot var(md) - md \cdot var(lo1) \end{aligned}$$

$$\begin{aligned} \partial cov(lo1, md_{\gg 1}) &= 0.5 \cdot cov(lo1, hi1) \cdot hi2 - cov(hi1, md_{\gg 1}) \cdot lo1 - 0.5 \cdot cov(lo1, md_{\gg 1}) \cdot lo1 - cov(lo1, md_{\gg 1}) \cdot lo2 + cov(md, md_{\gg 1}) \cdot lo1 + 0.5 \cdot cov(md, md_{\gg 1}) \cdot lo2 - 0.5 \cdot cov(lo1, hi1) \cdot md_{\gg 1} + 0.5 \cdot \\ &cov(lo1, hi2) \cdot hi1 + 2 \cdot cov(lo1, lo2) \cdot hi2 - 0.5 \cdot cov(hi2, md_{\gg 1}) \cdot lo1 + 0.5 \cdot cov(lo1, lo2) \cdot lo1 + 2 \cdot cov(lo1, hi2) \cdot lo2 - cov(lo1, lo2) \cdot md_{\gg 1} + 0.5 \cdot cov(lo2, md_{\gg 1}) \cdot md - cov(lo1, hi2) \cdot md_{\gg 1} - 1.5 \cdot \\ &cov(lo1, md_{\gg 1}) \cdot hi1 - 1.5 \cdot cov(lo1, md_{\gg 1}) \cdot hi2 + cov(lo1, md_{\gg 1}) \cdot md + 0.5 \cdot lo2 \cdot var(lo1) - 0.5 \cdot md_{\gg 1} \cdot var(lo1) \end{aligned}$$



...

# Writing Models Compositionally

- Embedded chemical notation

Programs freely contain both chemical reactions and control flow  
Can generate unbounded-size reaction networks

- Rich data types

*numbers, species, functions, networks, lists, flows (time-courses)*

*flows* are composable functions of time used in *rates, plotting, and observation*

- Modern abstractions

*Functional:* programs take *data* as parameters and produce *data* as results

*Monadic:* programs also produce *effects* (*species, reactions, liquid handling*)

*Nominal:* *lexically scoped* chemical species (species are not "strings")

# Ex: Predatorial

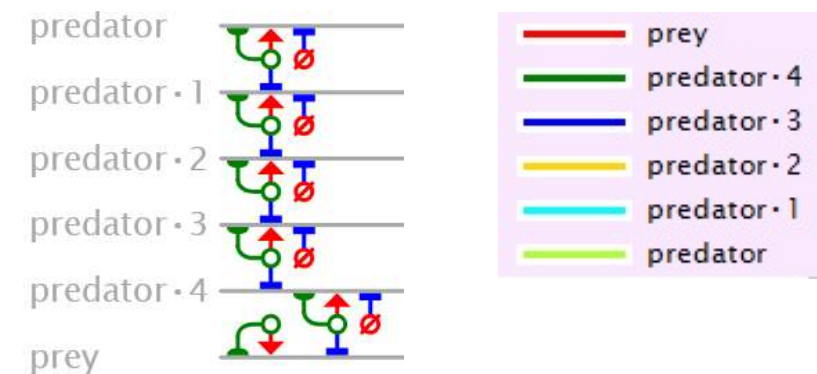
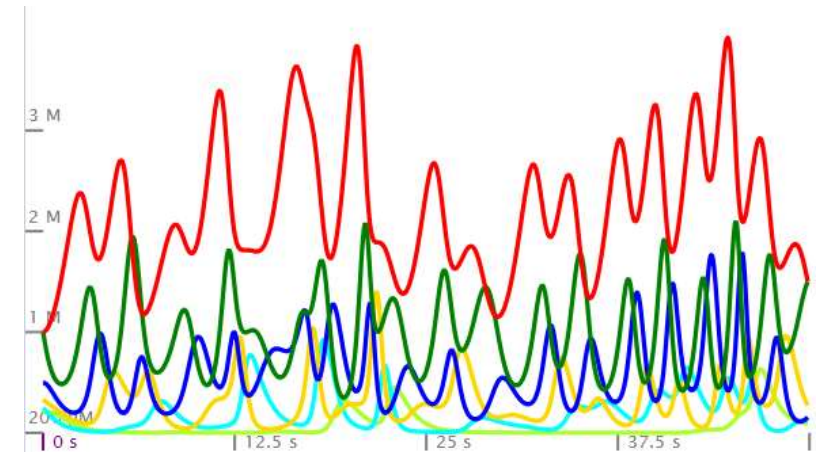
```

function Predatorial(number n) {
  if n = 0 then
    define species prey @ 1 M
    prey -> 2 prey // prey reproduces
    report prey
    yield prey
  else
    define species predator @ 1/n M
    species prey = Predatorial(n-1)
    prey + predator -> {n} 2 predator // predator eats
    predator -> ∅ // predator dies
    report predator
    yield predator
  end
}

species apexPredator = Predatorial(5)
equilibrate for 50
  
```

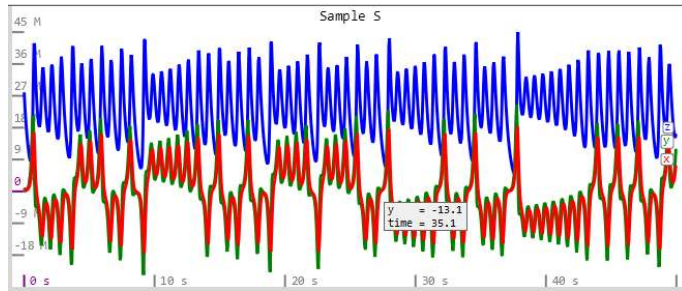
```

//=====
// Creates a stack of predator-prey
// relationships in Lotka-Volterra style,
// and returns the apex predator.
//=====
  
```



# Mass Action Compiler

- Lorenz chaotic attractor

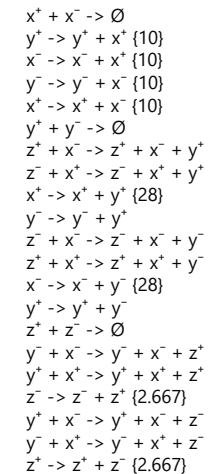
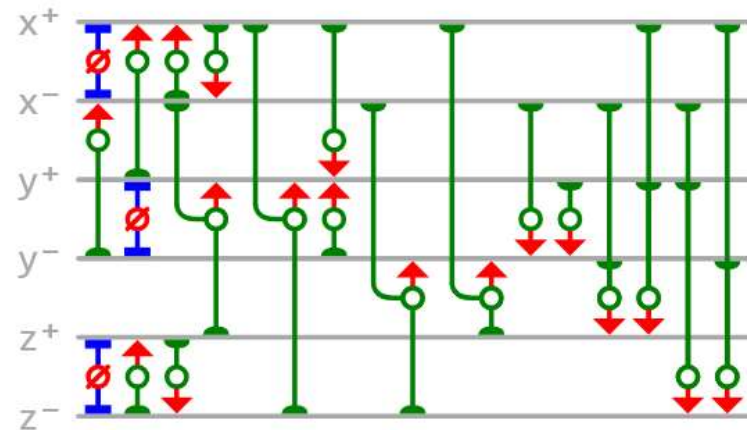
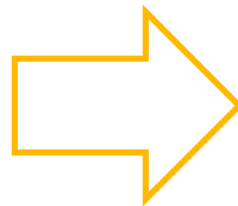


not mass action

$$\begin{aligned} \partial x &= s \cdot y - s \cdot x \\ \partial y &= r \cdot x - x \cdot z - y \\ \partial z &= x \cdot y - b \cdot z \end{aligned}$$

$s = 10$   
 $b = 8/3$   
 $r = 28$   
 $x_0 = 1$   
 $y_0 = 0$   
 $z_0 = 28$

Hungarian criterion violation



Initial:  
 $x^+ = 1$   
 $x^- = 0$   
 $y^+ = 0$   
 $y^- = 0$   
 $z^+ = 28$   
 $z^- = 0$

<= Demo: LorenzAttractor

# References

## **Experimental biological protocols with formal semantics**

Alessandro Abate, Luca Cardelli, Marta Kwiatkowska,  
Luca Laurenti, Boyan Yordanov. CMSB 2018.

## **Kaemika app - Integrating protocols and chemical simulation**

Luca Cardelli. CMSB 2020.

## **Kaemika User Manual**

<http://lucacardelli.name/Papers/Kaemika%20User%20Manual.pdf>

## Integrated modeling

Of chemical reaction networks and protocols

How the Kaemika app supports it

Why it needs a *new language* for smooth integration

## Closed-loop modeling, experimentation and analysis

For complete lab automation

To “scale up” the scientific method

## Thanks to:

Gold (parser generator)

OSLO (ODE simulator)

C#/Xamarin (IDE)

App store reviewers

## NO thanks to:

XAML (general obfuscator)

App store certificates

Dark mode support

# Conclusions



# Chemical reaction networks

- A fun language to program with
- Compilable to real molecules
- Executable "in your kitchen"
  
- Still relative primitive, we need to build more programming abstractions